Determination of the Critical Slip Surface

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ABSTRACT

The critical slip surface is important in determining the stability of a slope. There are various methods to determine this surface. Many of the methods are only applicable in very specific conditions. The simplest method is to assume a circular slip surface. This may be a good assumption in some conditions, but where different soil layers with different strengths are present this may not prove to be accurate. Of the various non-circular methods, only the Force-Equilibrium procedures are suitable for hand calculations. If computers, then, must be used to determine the slip surface, a very accurate method should be used. While the most accepted methods are "good enough", perhaps using newer mathematical algorithms to determine the critical slip surface would provide a more accurate representation of it. These new mathematical algorithms, classified as meta-heuristic algorithms, require a computer and some intensive calculations to determine the effects of one particle on others. This, in essence, provides the "path of least resistance", or the weakest surface within the soil where failure is most likely to occur. Using a meta-heuristic algorithms such as the Monte Carlo method, simulated annealing method, genetic algorithm, or particle swarm optimization, a more accurate representation of the critical slip surface can be determined. These methods are explained and reviewed to provide a better understanding of the strengths and weaknesses of each.

INTRODUCTION

Determining the factor of safety against slope failure is an important aspect of Geotechnical engineering. There are many different methods to determine this factor of safety. The majority of these methods require an estimation of the location of the slip surface. The location of the slip surface is the major factor determining the factor of safety. The location of the critical slip surface depends greatly on the in-situ conditions. While it is impossible to know exactly what the in-situ conditions are at every location, a good knowledge of the conditions is generally sufficient. Due to this it is impossible to determine the actual critical slip surface in the field, however the slip surface corresponding to a theoretical minimum factor of safety can be determined.

The most basic slip surface is a straight line. This is used for the case of an infinite slope. The condition of an infinite slope is, for obvious reasons, not realistic. The infinite slope is, however, a good assumption for very long slopes with a relatively shallow slip surface.

Another assumption for the slip surface is a circular arc. The only input needed to define this surface is the center point and the radius of the circle. This method has easy solutions using basic trigonometry. A circular arc is, in some cases, a good approximation, but not in all cases. The circular arc is similar to the log spiral, however the log spiral requires charts or complicated mathematics to define the slip surface. Both assume a smooth slip surface. Once a circular slip surface is estimated, a computer can systematically determine the minimum factor of safety by varying the location of the center point and the radius of the circle. This is a very quick method to determine the critical slip surface when a circular failure is likely to occur in the field.

Another method of determining the critical slip surface is to use finite element analysis. This analysis requires no primary assumption of the critical slip surface, instead a mesh is defined which describes the soil characteristics and the critical slip surface is found during the analysis. The only problem with finite element analysis is that the analysis may become "stuck" in a local minima which causes the global minima to increase. While this method deserves more consideration, and will receive it from Mahab Khatib, it outside the scope of this paper.

The purpose of this paper is to discuss advanced methods of determining the critical slip surface. A surface that can assume any generally concave shape is the best assumption for the critical slip surface. Included within this assumption would be the circular arc surface and the log spiral surface. If this surface can be determined systematically using a computer it would be a more efficient method than iterating by hand. The computer would provide a more comprehensive analysis to find the lowest factor of safety.

UTILIZING THE COMPUTER

The main reason that circular slip surfaces have become so popular, and have been used for so long, is their ease of use. When the methods of determining the critical slip surfaces were being published (1950s through 1960s) the computer was not very accessible. Large computers (800 square feet) had the processing power to compute a few hundred to a thousand cycles per minute. This is much faster than pencil and paper, however each iteration would require several thousand cycles. Besides requiring a large amount of time to solve complicated problems, these computers were rare, and were used for more important things like calculating trajectories for missiles that would never be used. It was difficult to get time on one of these machines. Even if time could be had, the programming had to be done on punch cards. Each instruction punched onto a thick paper card that would tell the computer what to do. These cards basically translated into assembly language or machine code. All of this was a very drawn out process. It would have been impossible to determine the stability of every highway embankment using these computers.

The circular methods were developed so as to enable the engineer to get a "good" estimate of the minimum factor of safety of the slope using a slide rule and some graphing paper. In many cases the circular failure surface is a good approximation of the actual failure surface. Close enough to use a slide rule. Many of the methods had some empiricism built in, so the minimum factor of safety would still include some conservatism. The assumption of a circular slip surface allowed the engineers the ability to iterate a few times and find the minimum factor of safety of a slope within a few days. This could be done for every embankment.

There have been some improvements in computers since the 1960's. In fact the calculator used by the average college student is more powerful than the early computers. Imagine that 800 square feet transformed into a small box that fits in the palm of the hand. Modern day laptop and desktop computers are infinitely more powerful than the old ENIAC computers, the "super computers" of their day. A "super computer" today has the capability of computing over 360 trillion instructions per second; while a desktop computer can handle 100 million. Even the desktop computer is more than capable of calculating the minimum factor by iterating over hundreds of circular failure surfaces in less than a second.

The idea, however, is to be able to compute a non-circular slip surface that provides the minimum factor of safety. Theoretically this could provide a lower factor of safety than the circular in many cases; it may even find the slip surface to be circular in other cases. One method of computing this is by iterating over all the possibilities. To do this a slope would be divided into n number of slices (see Figure 1), and the maximum and minimum depths of the surface at each slice would be defined. These bounds should correspond to the maximum and minimum possible depths of the slip surface. The points where the slip surface meets the surface should move along the surface instead of vertically. After this occurs one would select a step distance, and each point within the bounds would be checked for a minimum factor of safety, with the computer iterating over the step distance. The main problem with this "brute force" approach is optimization. A large slope with many slices could take weeks or months to compute. Far too long.

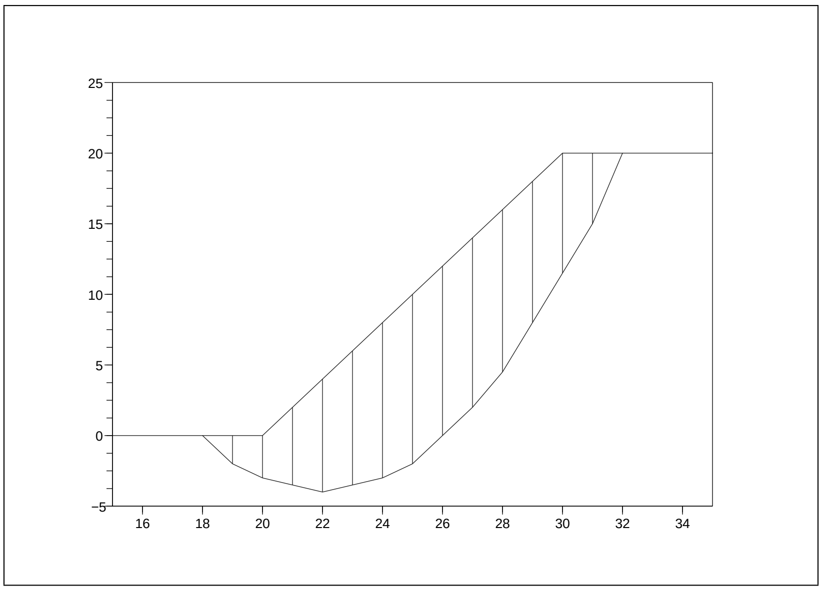

FIG. 1. Slope with Initial Failure Surface and Slices

A better method must exist that would more efficiently find the minimum factor of safety, and still allow the computer to iterate along different failure surfaces to determine the most critical. This method must allow the computer to learn from previous iterations which surfaces are the most critical and combine them in some form to determine another set of critical surfaces and continue this process until a minimum factor of safety has been reached. Perhaps this fast searching capability will be found in meta-heuristic algorithms.

THE META-HEURISTIC ALGORITHM

Meta-heuristics, computer code which imitates some sort of natural phenomenon, have become more and more popular over the last decade. The main feature of meta-heuristic algorithms is that they allow the computer to "learn." The computer is programmed to try to solve a problem in different ways. The computer then evaluates each new method and the best methods are kept and improved upon. A good example of this is a robot walking.

The robot is not programmed to walk, however it is given basic instruction on how to walk and then told to reach point B. When the computer fails to walk it tries minor variations on the technique it used previously and this continues until the robot is able to reach point B. By this time the computer has "learned" to walk.

There are many methods of the meta-heuristic algorithm. These include the simulated annealing method, the genetic algorithm, taboo search (human memory), harmony search algorithm, the Monte Carlo method, and the particle swarm optimization, which includes groups searching for food. The methods which have been used for determining the critical slip surface are the Monte Carlo method, the particle swarm optimization method, the simulated annealing analysis, and the genetic algorithm (Cheng et al. 2008).

The use of algorithms which imitate natural phenomena will provide a more efficient method of determining the critical slip surface than brute force alone. The search optimization occurs through the use of random variations in the estimated slip surface until the critical surface is found. The different meta-heuristic algorithms apply these random variations in different ways to find the global minimum. At present only algorithms used for other purposes have been utilized. There have not been any new algorithms created specifically for the purpose of finding the critical slip surface. It is likely that any new algorithm will be created as a variation of an exiting method. The existing methods should be analyzed to determine their strengths and weaknesses, and their ability to find the critical slip surface.

Meta-heuristic algorithms are probabilistic algorithms. Deterministic algorithms are those which given the same initial conditions will produce the same outcome by the same manner. Probabilistic algorithms are not deterministic. Probabilistic algorithms, given the same initial conditions, will almost never produce the same outcome. The outcomes will, however, be good approximations of each other.

Using meta-heuristic algorithms to determine the slip surface require the use of a method to determine the minimum factor of safety. Any method can be used which computes the factor of safety from an assumed slip surface; from the General Method of Slices (Duncan and Wright 2005) to Morgenstern and Price (1965) to Spencer (1967). The two latter methods mentioned are the most popular.

Monte Carlo Method

The Monte Carlo method consists of two different methods, walking or jumping. Both employ random variations in the proposed critical slip surface to find a minimum factor of safety. This method has been in use since the early 20th century. Its use as a method of determining the critical slip surface was discounted in the late 1980s as it was not as efficient as deterministic methods.

This analysis of the Monte Carlo technique is based on Greco (1996), a walking method. Greco modified the old Monte Carlo technique to include the minimum factor of safety as a qualifying criterion. This analysis requires the same initial conditions described in Figure 1. The initial slip surface (S^o) is defined, and new, feasible slip surfaces ($S^1, S^2, ...$) are randomly derived from from S^0, by varying the location of each point an any direction, such that the factor of safety decreases. When the factor of safety has reached its lowest point (ie. the factor of safety of subsequent surfaces begins to increase) the critical slip surface has been obtained.

Whereas the brute-force method will test every possible slip surface within the set boundaries, the Monte Carlo method causes the search to stop once a minimum is perceived. This is a much more efficient method than brute force in terms of computing time. The major problem induced by this technique is the possibility that the surface will get stuck in a local minima. If the slope takes a shape that passes into a weak point in the soil it is possible that the factor of safety will increase slightly before leaving the local minima. The Monte Carlo method causes the search to stop when an increase is encountered. The global minimum likely exists outside of this local minimum. There have been some changes to the Monte Carlo method by Abdallah et al. (2001) which help in alleviating this problem.

In general the Monte Carlo method can be characterized as a brute force technique with a computed termination criterion which greatly reduces the required computing time. This is an easy to program solution to determining a non-circular critical slip surface.

Simulated Annealing Method

The simulated annealing method, as presented by Cheng (2003), solves a global optimization problem through the random evaluation of an objective function. This occurs in such a way that leaving a global minimum is not difficult. The method was developed to simulate the crystallization of molten solids. This method searches for the minimum energy level of the system, the point at which the minimum value of the objective function no longer changes over various iterations.

After the geometry of the problem has been defined the method proceeds as follows:

1. Initial vertical locations of the entrance and exit locations of the slip surface are defined.
2. Maximum and minimum horizontal bounds of the entrance and exit of the slip surface are defined.
3. The horizontal locations of the interior nodes are defined
4. Initial vertical locations of the interior nodes are defined.
5. Bounds are placed on the vertical location of the interior nodes.

If the entrance location is point A and the exit point B, with the interior points C,D,E,F, then the control variables are defined as $(A_x, B_x, C_y, D_y, E_y, F_y)$. The annealing method iterates through the variables sequentially. First the A_x and B_x variables are varied sequentially through the bounds defined by the engineer. Once these values are determined the interior y variables are varied sequentially searching for the minimum factor of safety. Once a minimum has been achieved the process repeats by varying A and B then varying B to F. As the step size between iterations tends to 0 the ability of the simulated annealing method to find the actual critical slip surface increases. As the step size tends to zero the time required for computation tends to infinity. This is a problem and the algorithm should be programmed to stop after the reduction in the factor of safety reaches a user specified limit.

The simulated annealing method is similar to the Monte Carlo method, testing every possible solution with some defined bounds. The annealing method doesn't use random variations to find the minimum factor of safety. In this way it is similar to the brute force method. By fixing one dimensional aspect of each point this method greatly reduces the number of iterations required for searching. An experienced engineer should be able to estimate very closely where the location of the entrance and exit of the critical slip surface will be located, and can thus limit the boundaries of these points. Since the interior iterations occur for each iteration of the location of A and B, reducing the possible locations of A and B results in a significant decrease in computing time, and will likely result in a more accurate prediction of the critical slip surface and the corresponding minimum factor of safety. The simulated annealing method is more difficult to program than the Monte Carlo method, but is still relatively simple. For the increase in accuracy and processing speed the simulated annealing method is worth the extra effort.

Genetic Algorithm

Zolfaghari et al. (2005) presents the use of the genetic algorithm for determining the critical slip surface. The genetic algorithm mimics evolutionary biology. Given numerous variations of the same species, the one best suited for the environment has more probability of reproducing. These variations are random, based on the better examples of the population. This algorithm refers parents as the initial population and the children as the iteration, then the children become parents.

The genetic algorithm uses "chromosomes" to store the characteristics of each possible solution in binary format. Each chromosome is rated with a fitness level and the chromosomes are ordered with the best fitness level first. In order to ensure the best solution carries on, the best chromosome is copied to the next generation. Half of the remaining population is allowed to reproduce. This occurs by randomly selecting portions of the parent chromosome to transfer to the child. Minor variations (mutations) in the chromosomes are induced to a small percentage of the population during the reproduction process in order to avoid or escape from local minima. When the reproduction process is complete the children are evaluated according to their fitness value, ordered, and the process repeats.

Applying this algorithm to find the critical slip surface we fill the chromosome with information about the slip surface, specifically the horizontal location of the beginning of the slip surface and α at each slice intersection. One chromosome contains an entire slip surface definition. It must be checked to make sure that the surface defined is feasible. If it is, the factor of safety (fitness value) is calculated. Two new chromosomes (slip surfaces) are created from the parents. The parents do not leave the population until their fitness value falls within the lower half. This requires that the initial population be computed from an original with small variations, or no chromosomes are eliminated until a defined population size is reached. The iterations continue until reproduction has occurred a certain number of times or until subsequent generations are no fitter than previous generations, indicating that the slip surface has reached a minimum factor of safety.

The genetic algorithm has the ability to learn which characteristics produce lower factors of safety and combine them in order to quickly find the critical slip surface and the global minimum factor of safety. This ability causes it to be more efficient than the other methods previously discussed. The possibility of not being able to leave a local minima is higher using the genetic algorithm than with the simulated annealing method, however it is still quite small. The genetic algorithm will find the correct answer much faster when a better guess of the critical slip surface is used. This is not the case with the simulated annealing method as all possible outcomes within the bounds are calculated.

The introduction of random variations causes the final result to differ slightly from run to run, so the actual critical slip surface is not obtained; only a good approximation of its location is obtained. The genetic algorithm allows for fast searching through creating a more fit population. Sometimes the iterations have to go several generations to improve upon the previous best, so it is important to make sure the termination criteria allow for this, and don't prematurely end the searching process.

Particle Swarm Optimization

There are various algorithms included in the Particle Swarm Optimization category. These typically model the feeding patterns of different animal species. One application of particle swarm optimization in determining the critical slip surface is by Cheng et al. (2008), where a fish searching for food algorithm is used. The specific algorithm used is the Artificial Fish Swarms Algorithm. This algorithm seeks to describe a school of fish searching for food. The fish in the school learn from the "best fish", or the fish with the lowest objective function, most food in nature, lowest factor of safety in slope stability. This is an evolutionary algorithm similar to the genetic algorithm.

The artificial fish swarms algorithm requires an initial pool of N possible slope surfaces. These would be generated randomly in accordance with Cheng et al. (2007). This seems to be a good method to computationally guess at the critical slip surface, especially if many surfaces should be created. After a pre-determined number of slip surfaces are created they are analyzed by the algorithm and sorted by the factor of safety. The lowest factor of safety becomes the "best fish". Random numbers determine which slip surfaces, or portions of the surfaces, should change locations. They change based on the factor of safety of "better fish" nearby and on the relative distance to the "best fish". After the surfaces have changed locations once, the factors of safety are recomputed, each surface sorted according to the lowest factor of safety, and the "best fish" is found. This process is iterated until a termination criteria has been met. In most particle swarm optimization algorithms the only termination criterion is the number of iterations. Cheng et al. (2008) proposes that the termination criteria be based on a set number of iterations where the lowest factor of safety does not change. This criteria provides better results than simply guessing how many iterations will be required.

There are some possible problems with the artificial fish swarms algorithm. Using a set number of surfaces it is possible, though not highly likely, to miss the global minimum. The chance of this occurring using this algorithm is greater than with the genetic algorithm or simulated annealing method. Based on studies by Cheng, it appears the artificial fish swarms algorithm performs better in complicated soil conditions than the other meta-heuristic algorithms. It also appears to handle sharp contrasts in soil layers (alternating hard and soft layers) better than the others. Of the methods discussed this is possibly the most difficult to program, with the genetic algorithm trailing closely. This algorithm, however, will likely out-perform the other methods with respect to the speed of computation. It will not definitely out-perform the others, as the number of "fishes" or slip surfaces that move is somewhat random. The factors of safety will have to be computed for each surface that moves; if many surfaces move the required processing time will increase.

Cheng et al. (2008) compared the artificial fish swarms algorithm with the genetic algorithm using the examples solved in Zolfaghari et al. (2005). The factor of safety for the artificial fish swarms algorithm is lower, indicating that a more critical slip surface has been obtained. In comparing the artificial fish swarms algorithm with his earlier work with the simulated annealing method, Cheng finds the fish swarms algorithm to provide a lower factor of safety, thereby better predicting the critical slip surface.

CONCLUSIONS

While these meta-heuristic algorithms provide a more accurate prediction of the critical slip surface, the difference in the factors of safety between them and the circular or log spiral slip surfaces are small. In simple one or two layer homogeneous soil profiles there is almost no improvement. The major improvement comes in soils with many layers and complicated flow mechanisms that cause failure along very non-circular surfaces. The use of these meta-heuristic models is likely only useful for complicated soil profiles, where a very accurate factor of safety is needed. In mining, dredging, or cut slopes for other purposes, a very accurate critical slope surface is required. In some conditions it will be very economical to conduct a meta-heuristic analysis. In most cases, however, assuming a circular surface using the Spencer or Morgenstern and Price method, and iterating over a radius and entry/exit locations will generally provide a critical slip surface that is "good enough". Many of the meta-heuristic algorithms could be implemented to more rapidly find the critical slip surface using a circular surface as well. This, perhaps, is good enough for basic commercial software. Complicated analysis should be left for the experienced engineer, who will know better than a computer what is a likely slip surface, and will be able to program a meta-heuristic model capable of searching out the minimum factor of safety. These models may be quicker to use in three dimensional analysis than finite element models, which can take weeks or months to run, depending on the discretation intervals.

In all the papers discussing these meta-hueristic methods no specific case studies have been conducted or even mentioned where a meta-heuristic analysis was used to calculate the critical slip surface and the minimum factor of safety. They have been compared with other theoretical soil models, but no real-life examples. The actual soil conditions are often heterogeneous, and more complicated than can be obtained with the subsurface investigation techniques currently utilized. A great computer program is only as good as the knowledge put into it. The older models have some empiricism built-in, which accounts for experience; this experience cannot be obtained using purely theoretical models when the details of the situation are not completely understood.

Once technology allows for a complete understanding of the site conditions, and science allows for a complete understanding of the reaction of soil to the environment around it, we will be able to use theoretical models to accurately predict future events. Until then our best analysis and probabilistic models will have to do.

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